Workshop Report

SOPAC WORKSHOP ON SEA AND SWELL DATA
21 November - 5 December 1989
Rarotonga, Cook Islands

by
Russell Howorth*
Training Co-ordinator, Techsec

Prepared for: South Pacific Applied Geoscience Commission (SOPAC) Training Programme

Contributed by: The Government of Norway and *ESCAP/UNDP RAS/86/125 Development of South Pacific Institutional Capability in Marine Minerals & Technology (CCOP/SOPAC)
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>3</td>
</tr>
<tr>
<td>Funding</td>
<td>4</td>
</tr>
<tr>
<td>Participants</td>
<td>4</td>
</tr>
<tr>
<td>Content of Course</td>
<td>5</td>
</tr>
<tr>
<td>Budget</td>
<td>6</td>
</tr>
<tr>
<td>Participants Assessment/Examinations</td>
<td>6</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>6</td>
</tr>
</tbody>
</table>

### Appendices

1. [Remarks at Opening and Closing of Workshop] .......... 7
2. Workshop Proposed Programme                    11
3. Actual Daily Schedule                          13
4. Examination Paper                             17
INTRODUCTION

The SOPAC Workshop on Sea and Swell Data was held at Te Atukura, the Government Reception Centre, Rarotonga, Cook Islands from 21 November to 5 December 1989. The Workshop was part of the wave energy programme being carried out by SOPAC in conjunction with, and sponsored by the Government of Norway.

The Workshop Opening Ceremony was held at 10.00 on 21 November, Mr A Utanga, Secretary of Internal Affairs presided. Welcoming remarks were presented on behalf of the Government of the Cook Islands by Mr Stuart Kingan and for SOPAC by Dr Russell Howorth, Training Co-ordinator (Appendix 1). The Workshop was officially opened by Deputy Prime Minister and Minister of Works and Energy, Mr Inatio Akururu.

The Government of Norway has recently agreed to extend and expand its current wave rider buoy programme in the South Pacific with SOPAC to include Fiji, Kiribati, Tuvalu and Vanuatu, in addition to Cook Islands, Tonga and Western Samoa.

As part of this programme, training will be provided to island nationals already employed by governments to act as local supervisors for the country in their wave energy programme. This Workshop is aimed at providing initial training for two nationals each from the Cook Islands, Fiji, Kiribati, Tonga, Tuvalu, Vanuatu and Western Samoa. (Unfortunately Vanuatu was unable to attend). This initial training provided an introduction to sea and swell wave theory, the operation of wave rider buoys and the collection and analysis of data.

The Course Instructors were Professor Johannes Falnes from the University of Trondheim and Dr Stephen Barstow from OCEANOR, Trondheim. They were assisted by Dr Russell Howorth SOPAC, Training Co-ordinator and Dr Ralf Carter, SOPAC Marine Scientist and Mr Egil Olsen from OCEANOR.

At the end of the Workshop, an examination was held.

The Workshop Closing Ceremony is recorded in Appendix 1.
### FUNDING

The Workshop was funded by the Government of Norway. In-kind support was provided by the Cook Islands Government and the UNDP Project RAS/86/125 within SOPAC.

### PARTICIPANTS

A total of 17 participants attended the Workshop: they were from 6 countries in the region: Cook Islands (7), Fiji (2), Kiribati (1), Tonga (3), Tuvalu (2) and Western Samoa (2).

The participants were as follows:-

<table>
<thead>
<tr>
<th>Name</th>
<th>Country</th>
<th>Position</th>
<th>Office</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atatoa Herman</td>
<td>Cook Islands</td>
<td>Assistant Harbour Engineer</td>
<td>Harbour Engineer's Office</td>
<td>Ministry of Trade Labour &amp; Transport P.O.Box 61, Rarotonga</td>
</tr>
<tr>
<td>Tenga Mana</td>
<td>Cook Islands</td>
<td>Civil Engineering Technician</td>
<td>Ministry of Works</td>
<td>P.O.Box 102, Rarotonga</td>
</tr>
<tr>
<td>Nooroa Parakoti</td>
<td>Cook Islands</td>
<td>Engineering Officer</td>
<td>Harbour Engineer's Office</td>
<td>Ministry of Trade Labour &amp; Transport P.O.Box 61, Rarotonga</td>
</tr>
<tr>
<td>David Ngatupuna</td>
<td>Cook Islands</td>
<td>Engineering Technician</td>
<td>Ministry of Works</td>
<td>P.O.Box 102, Rarotonga</td>
</tr>
<tr>
<td>Tereapii Timoti</td>
<td>Cook Islands</td>
<td>Assistant Electrical Engineer</td>
<td>Electric Power Supply</td>
<td>P.O.Box 112, Rarotonga</td>
</tr>
<tr>
<td>Alan Utanga</td>
<td>Cook Islands</td>
<td>Civil Engineering Technician</td>
<td>Ministry of Works</td>
<td>P.O.Box 102, Rarotonga</td>
</tr>
<tr>
<td>Tony Utanga</td>
<td>Cook Islands</td>
<td>Secretary Internal Affairs</td>
<td></td>
<td>P.O.Box 98, Rarotonga</td>
</tr>
<tr>
<td>Rajesh Chandra</td>
<td>Fiji</td>
<td>Senior Power Systems Planner</td>
<td>Ministry of Energy</td>
<td>Private Mail Bag, Suva</td>
</tr>
</tbody>
</table>

[MR92 - Howorth]
CONTENT OF THE COURSE

The outline of the course is given in Appendix 2. The actual daily schedule is given in Appendix 3. The normal daily work programme was 08.30 - 10.00; 10.30 - 12.00; 13.30 - 15.00 and 15.30 - 17.00.
BUDGET

The approved draft budget for the Workshop was F$50,464. A statement of expenditure will be provided later to the donor once accounting has been completed.

PARTICIPANTS ASSESSMENT/EXAMINATIONS

Work was continually reviewed during the Workshop with questions. The University of Trondheim officially recognised the Workshop as an examination based continuing education course entitled "Measurement of Ocean Waves". Participants were not obliged to sit the examination at the end of the Workshop. A copy of the examination paper is given in Appendix 4.

16 Participants sat the exam.

ACKNOWLEDGEMENTS

Many people provided invaluable assistance which contributed to the success of the Workshop. In particular;

Ms E. Kotekau for secretarial assistance;
Mrs K. Brown, Manager of Government Protocol and Reception premises, and the staff of Te Atukura for preparing refreshments;
Staff of Cook Islands Government for photocopying facilities.
APPENDIX 1

Opening Remarks by Cook Islands National Representative to SOPAC
Mr Stuart Kingan

Welcome to Professor Falnes who has come all the way from Norway to conduct this Workshop. Also I welcome in advance of his arrival Dr Steve Barstow also from Norway.

Welcome to all our participants from other Pacific countries and to those from the Cook Islands.

This is a very important Workshop on the dynamics of ocean waves which Norway is financing as part of our SOPAC Wave Measuring Programme in the Pacific.

Norway is recognized as the world’s leading country in the practical development of wave energy.

This Workshop is not only the first of its kind in the Pacific - I have been told by the Professor this morning that it is the first of its kind in the world.

I now have much pleasure to call upon Dr. Russell Howorth, Training Co-ordinator of SOPAC to make brief comments on behalf of the regional organization following which, I will invite the Hon. Minister of Energy and Deputy Prime Minister, Mr Inatio Akaruru, to formally open this Workshop.
Introductory Remarks by the SOPAC Training Co-ordinator
Dr Russell Howorth

Honorable Deputy Prime Minister Akaruru, participants and guests, the Director of SOPAC, Mr Jioji Kotobalavu has asked me to welcome you here today for the opening of this Workshop on Sea and Swell Data. Let me first say that at its recent annual meeting CCOP/SOPAC agreed to change its name to South Pacific Applied Geoscience Commission, but retain SOPAC as its acronym.

Now let me give you a little background to this Workshop. The Government of Norway has recently agreed to extend and expand its current wave energy programme in the South Pacific with SOPAC. Under this new agreement the number of wave rider buoys deployed will be increased. At present there are buoys in Cook Islands, Tonga and Western Samoa, shortly there will be buoys available for deployment in Fiji, Kiribati, Tuvalu and Vanuatu. An expert from Norway will shortly arrive in the region for up to a 2 year period to oversee the wave energy programme, and an island national will be recruited to Techsec as a graduate trainee to be trained as a local counterpart.

Training will also be provided to other island nationals already employed by governments to act as local supervisors for their countries wave energy programme. This Workshop is aimed to provide the initial training for two nationals from each of Cook Islands, Fiji, Kiribati, Tonga, Tuvalu, Vanuatu and Western Samoa. (Unfortunately Vanuatu has been unable to attend). This initial training will provide an introduction to sea and swell wave theory, the operation of wave rider buoys and the collection and analysis of data.

SOPAC and in particular the member countries participating in this Workshop are grateful to the Government of Norway for providing the funding for the Workshop as part of its extended wave energy programme. In kind support has also been provided by the Cook Islands Government and the UNDP Project within SOPAC.

We are also grateful to Professor Falnes of the University of Trondheim, and Dr. Stephen Barstow of OCEANOR, also based in Trondheim for giving their time to the preparation and presentation of the Workshop.
Opening Address by Deputy Prime Minister 
and Minister of Energy 
Mr Inatio Akaruru

Firstly, I must welcome all our visitors to Rarotonga and hope your stay here will not only be productive but also enjoyable.

I welcome Professor Falnes and in advance of his arrival Doctor Barstow who have travelled all the way from Norway to conduct this Workshop. I welcome Dr Russell Howorth, Training Coordinator of SOPAC who has organised this Workshop and Dr Ralf Carter who has co-ordinated the wave monitoring programme in the Pacific for SOPAC.

And importantly, I welcome the Workshop participants from Fiji, Kiribati, Tonga, Tuvalu and Western Samoa, also of course, our own Cook Islands participants.

All these Pacific countries share a major common problem, that of the reliance on imported oil as the main source of energy. Not only is imported oil very expensive, particularly when it has to be shipped in small quantities over long distances, but its storage is difficult and its supply often erratic and uncertain.

Our Government realises that it is not only possible but very desirable that we harness the sources of locally available energy, in order to eventually replace oil imports.

We are starting to make considerable use of our abundant solar energy and continue to carry out research into the potential for wind energy. We are keeping up with overseas development in ocean thermal energy conversion. But we feel that eventually the ocean waves will become one of our major energy sources on major islands like Rarotonga.

Thanks to the support we have received from the Government of Norway and from SOPAC we have been conducting a programme into monitoring the wave energy, off the south east of Rarotonga for several years.

So far these show great promise, and with the technical advances made by Norway in the development of successful wave operated power plants, we can see development in the not too distant future, which will go a long way towards solving the energy problem, faced by all countries represented at this Workshop.

I wish the Workshop every success, and it is my pleasure to declare this Workshop open.
Report of Closing Ceremony

The Wave Energy Workshop was formally closed by the Minister of Trade Labour and Transport, Vaine Taiirewa who is also responsible for SOPAC in the Cook Islands. The Workshop was closed on 4 December 1989 at 1515 hrs.

Mr Stuart Kingan, Cook Islands permanent representative to SOPAC said he was pleased to have been involved in holding one of the highest level workshops in the region; he congratulated the 16 participants who had done well at this workshop as Professor J Falnes was pleased with the results of the workshop. He further added that he is confident that wave energy when developed will contribute a lot to the requirements of small island governments within the region, and is reminded of the intention of the Norway Government to build a Pilot Wave Energy Station in the South Pacific so that its potential could be evaluated in the not too distant future.

The participant from Tonga, Mr Saimone Helu, on behalf of all the participants thanked the host government of the Cook Islands for agreeing to hold the Workshop in Rarotonga and for the many courtesies and hospitalities both at the workshop and outside of the workshop. He thanked the Government of Norway for funding the workshop and for providing the excellent tutors, Professor J Falnes and Dr S Barstow, who had enlightened all of the participants who had learned a lot about wave energy in the short time of the workshop.

Professor Johannes Falnes from the Technical University of Trondheim, Norway, thanked all those who made the workshop a success. He was particularly pleased that both he and Dr Barstow had been able to motivate all the participants to work really hard and come to grips with the lectures and learn about wave energy. He is pleased with the workshop and paid tribute and thanks to Dr Russell Howorth on behalf of SOPAC, Tony Utanga who was of great value to keep the workshop working well, and the Cook Islands Government for the excellent facilities and for welcoming the overseas participants and for being kind and friendly to them.

The Cook Islands Minister for Trade Labour and Transport, Mr Vaine Taiirea, apologised for not being able to be at the opening of the workshop but he made sure that he attended the closing. He was particularly pleased on behalf of the Cook Islands Government and its people for being able to host this important and high level regional workshop on Wave Energy. He asked what did it mean to learn about waves and went on to say that although the island states of the Pacific are small in land mass, their greatest asset is the great Pacific Ocean which the SOPAC region was learning to use for the development of Pacific peoples in, for example, energy, fisheries, etc. He was pleased to acknowledge Professor Johannes Falnes and Dr Stephen Barstow for coming a long way to run and tutor at the workshop.

He encouraged all participants when they return to their countries to disseminate the knowledge they had learned to their own people. He hoped every participant from overseas had enjoyed their stay in Rarotonga. He particularly thanked the Government of Norway for providing excellent tutors in Professor J. Falnes and Dr Barstow and for funding the project. He wished overseas participants a safe and pleasant journey back to their country, and declared the workshop closed.

The Reverend Papa Aratangi of the Cook Islands Christian Church gave a stirring message from the Gospel according to St Matthew, Chapter 8 verses 23 to 27, referring to the way Jesus calmed the waters during a storm on the sea of Galilee, and formally closed the Wave Energy Workshop with a prayer.
APPENDIX 2

Workshop Proposed Programme

Introduction

Background. The wave program/Norway. Course contents. Introduction to course participants/area of interest. Demonstration of ARGOS real time data.

Review of Wave Power Research

Energy demand. Wave energy resource. Wave power absorption considered as an interference phenomenon. Classification of wave energy converters (according to location, geometrical extension and orientation etc.) Various proposed and examined devices, Japanese, British, Norwegian, Swedish etc. (Masuda navigation buoy, KAIMEI, on-shore OWC, Salter duck, Bristol cylinder, the Clam device, Tapchan device, phase-controlled point-absorber buoy, hose-pump device etc.). Video: "From waves to electricity" Techno-economic assessments of wave-power devices.

Other Applications of Wave Data


Waves of the South Pacific

Simple introduction to the meteorology of the region (trade winds, cyclones, roaring forties). Wave parameters. Wave climate of the region. Waves (swell; wind waves, storm waves, tsunamis). Sources of information.

Linear Wave Theory


Wave Spectra

Real waves. Examples of wave records. Time series and zero-crossing analysis. Wave spectra (frequency spectrum and directional spectrum). Parameters from spectrum. PM spectrum. JONSWAP spectrum. Examples of real spectra

[MR92-Howorth]
Wave Measurements and Data Transmission


The ‘Waverider’ Buoy

How it works. What to look for. Deficiencies. Transfer function. WADIC correlations. Possible field trip/excursion?

Numerical Wave Model and Hindcast Data

Simple introduction to wave modelling. Sources of wave model data for the South Pacific (3G-WAM, BMO, N.Zealand Met.Service, GSOWM). Regional hindcast to extend measured wave data. Comparison between wave model and measurements.

Wave Statistics

Short-term statistics of single waves (wave groups, Rayleigh distribution, wave steepness, examples of H-T distributions). Long-term statistics (of e.g. Hs). Long-term data set based on hindcast and measurements (indicating e.g. climatical cycles). Extrapolation to design wave height. Joint distributions (scatter diagrams). Duration statistics.

Software for Handling of Data

ARGOS program (long-term series, and real-time registration demonstrated each day). Wave statistical packages. Demonstration of basic wave analysis (processing of time series of wave elevation to wave spectra and wave parameters). Production of scatter diagrams and other wave statistics. Demonstration of near-shore wave analysis by CREST/HISWA models. Participants will have the opportunity to try the various analysis themselves.

Repetition, Summary and Conclusion

(This may include comparisons based on real data from measurements, altimeter data, wave-model data and visual ship observations.)
APPENDIX 3

Actual Daily Schedule

Tuesday 21 November
1. Free
2(a) Opening Ceremony of the workshop.
(b) Introduction to the course.
3. Linear wave theory. (Regular wave. Circular orbits on deep water and elliptical orbits on shallow water. Phase velocity).
4. Linear wave theory. (Exercise followed by lecturing on the dispersion relationship with particular application to deep water and to shallow water).

Wednesday 22 November
1. Free
3(a) Review of wave-power research. (Videos on the Norwegian wave-power projects).
(b) Waves of the South Pacific. (Meteorology)
4. Exercises on Linear wave theory.

Thursday 23 November
1. Linear wave theory. (Superposition of regular waves. Standing wave. Group velocity, in particular for deep water and for shallow water.
2. Waves of the South Pacific. (Sources of wave data; visual and instrumented data).
3. Waves of the South Pacific. (Sources of wave data; remotely sensed data).
4. Exercises on Waves of the South Pacific and on Linear wave Theory.
### Friday 24 November
1. Waves of the South Pacific. (Discussion of yesterday’s exercises).
2. Waves of the South Pacific. (Sources of wave data; wave model data).
3(a) Waves of the South Pacific. (Sources of wave data; buoy measurements).
(b) Linear wave theory (Group and phase velocity. Refraction near shore. Wave spectrum.)
4. Exercise on Linear Wave theory and demonstration of and practice with computer software for presentation of processed wave data; data collected in Norway at 5 different locations were used.

---

### Monday 27 November
1. Linear wave theory (cont’d), waves having arbitrary direction of Bernoulli’s equation, hydrostatic and hydrodynamic pressure.
2. Waves of the South Pacific (cont’d), solution to exercise, comparison of various wave data for February 1989, discussion of two storms at Tonga.
3. Linear wave theory (cont’d), velocity potential. Laplace equation, linearised boundary condition, regular wave as particular solution.
4. Exercises in linear wave theory.

---

### Tuesday 28 November
1. Wave spectrum, example of three close frequency components and general case, moments of the spectrum, significant wave height and characteristic wave periods.
2(a) Wave spectrum continued.
(b) Exercises in wave power utilisation and demonstration of wave data on the portable Toshiba PC.
3. Wave measurement, introduction, the Waverider buoy, measurement principle, sources of error, buoy transfer functions.
4. Exercises and computer demonstration of wave statistics, time series from ARGOS and directional wave spectrum.

[MR92 - Howorth]
Wednesday 29 November
1. Linear wave theory, solution to problems.
2(a) Same as above.
(b) The Waverider buoy (cont’d)
3. Same as above.
4. Exercises in linear wave theory and computer demonstration of wave statistics.

Thursday 30 November
1. Videos showing wave measuring buoys in action from the WADIC and LEWEX experiments, WADIC summary.
2. Linear wave theory and wave power, fluid particle velocity of regular wave, and wave-power transport in regular and irregular waves, comments to exercises (questions) on wave-power utilisation.
3. Wave data transmission, discussion of the ARGOS satellite surveillance system.
4(a) Demonstration of computer connection by modern to Toulouse via Oceanor in Trondheim.
(b) Exercises in linear wave theory.

Friday 1 December
1(a) Excursion to Muri beach to observe the Waverider buoy at Rarotonga.
(b) Discussion of practical problems concerning the waverider buoy.
2. Continuation of above.
3. Solving exercises in linear wave theory.
4. Afternoon free for revision.

Saturday 2 December
Three hour examination in afternoon.

[MR92 - Howorth]
<table>
<thead>
<tr>
<th>Monday</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 December</td>
<td>Closure of Workshop</td>
</tr>
</tbody>
</table>
APPENDIX 4

Examination Paper
EXAMINATION

MEASUREMENT ON OCEAN WAVES

Saturday 2 December 1989 from 13:00 - 16:00

Material allowed: Approved calculator allowed

Contents:
Multiple Choice Questions (Problems 1 to 15) p. 2 to 11
 Calculation exercise (Problem 16) p. 12 to 13
 Copy of lecture-notes page (for information) p. 14 to 16
B: North Easterly, 315 degs.; South Easterly, 315 degs.; South Easterly, 135 degs.; Other.

C: Northerly, 0 degs.; Northerly, 360 degs.; Southerly, 360 degs.; Southerly, 180 degs.; Other.

D: South Westerly, 45 degs.; South Easterly, 360 degs.; North Easterly, 45 degs.; North Easterly, 225 degs.; Other.

2. Is there any difference to the answers to question 1 if the location had been Hawaii (in the northern Hemisphere) instead of Rarotonga?

Yes; No.

3. If the same question 1 had been asked with wind blowing in the direction of the arrow, would there be any difference to your answers?

Yes; No.
7. What is the approximate value of the average significant wave height (in metres) at a deepwater location near to your home island in the South Pacific?

1) Less than 1m  
2) 2-3 m  
3) Greater than 5m.

8. In which direction do the winds circulate around a low pressure system in the southern hemisphere? Is it as in diagram A or B?

1) A  
2) B

9. If the low pressure system in question 8 was in the northern hemisphere what would the answer be then?

1) A  
2) B
12. Swell waves are steeper (i.e., wave surface slope is greater) than wind sea waves.

True or False?

13. In the Waverider buoy the wave sensor measures the acceleration of the buoy as it moves up and down in the waves. The acceleration is twice integrated to give time series of wave elevation. This process introduces a transfer function (Fig. 2).

Are the following statements true or false?

Assume that this transfer function is the only source of error in the buoy measurement.

A) A regular wave with amplitude equal to 3m at 0.06Hz is recorded by the Waverider as a wave of 3.1m at 0.06Hz.

True or False?

B) A regular wave with wave period equal to 20 seconds and amplitude equal to 1m is recorded by the Waverider as a wave of amplitude approximately 0.92m.

True or False?
C) Regular waves with wave periods between 5 and 10 seconds are measured accurately by the Waverider with no reduction in the wave amplitude.

True or False?

14. The Waverider buoy can measure which of the following parameters:
   A) Wave Direction;
   B) Wave Spectrum;
   C) Significant Wave Height;
   D) Tm - 10;
   E) Wind Speed.

15. The ARGOS satellites are polar orbiting satellites. The Waverider measures significant wave height every third hour, i.e., 8 times a day. Is it true that the Waverider "sees" the satellites less than 8 times a day if it is moored at the equator?

Yes or No?
(d) Determine the new values (expressions or numerical values) for the wavelength $\lambda_1$, the phase velocity $c_1$, and the group velocity $c_g$, where the water depth has been reduced to $h_1 = 4\, \text{m}$. Assume that the shallow-water approximation is valid. (If numerical values are given, you should check whether this approximation is valid.)

(e) Determine the new value of the amplitude $a$, at this location (where $h = h_1 = 4\, \text{m}$).

Assume now that an array of units of a wave power plant has been put in place along a coast of length $B = 500\, \text{m}$ and that a fraction $\varepsilon$ of the incoming wave power is converted to electrical power.

(f) What is the electrical power $P_0$ if the converter units are floating devices moored in deep water, and $\varepsilon = \varepsilon_0 = 0.30$ ?

(g) What is the electrical power $P_1$ if the converter units are devices standing on the sea bottom in the water depth $h_1 = 4\, \text{m}$, and $\varepsilon = \varepsilon_1 = 0.20$ ?

In (f) and (g) use $\rho = 10^3\, \text{kg/m}^3$ or preferably $\rho = 10^3\, \text{kg/m}^3$ and $a = 9.8\, \text{m/s}^2$. 
This agrees with the previous result \( c_g = \frac{\omega}{k} \)

**Proof:** \( c = \frac{1}{T} = \frac{\omega}{k} \quad L = \frac{2\pi}{k} \)

\[
dL = -\frac{2\pi}{k^2} \, dk \\
dc = \frac{d\omega}{k} - \frac{\omega}{k^2} \, dk
\]

\[
c - L \frac{dc}{dk} = \frac{\omega}{k} - \frac{2\pi}{k} \frac{d\omega}{k} - \frac{\omega}{k^2} \frac{dk}{k^2} = \frac{\omega}{k} + \frac{d\omega}{dk} - \frac{\omega}{k} = \frac{d\omega}{dk} = c_g \text{ (q.e.d.)}
\]

We found \( \omega^2 = gk \left( \frac{a}{b} \right) \) (the "dispersion" equation)

On deep water \( (a/b = 1) \) \( \omega^2 = gk \Rightarrow 2\omega \, d\omega = g \, dk \)

Phase velocity \( c = \frac{\omega}{k} = \frac{g}{\omega} = \sqrt{g/k} \)

Group velocity \( c_g = \frac{d\omega}{dk} = \frac{g}{2\omega} = \frac{1}{2} \, c \)

Thus we have the important result that on deep water the group velocity is half of the phase velocity

On very shallow water \( (a/b = kh) \) \( \omega = k\sqrt{gh} \)

\( c_g = \frac{d\omega}{dk} = \frac{\omega}{k} = \sqrt{gh} = c \)

The group velocity and phase velocity are equal and independent of the frequency so long as the water may be considered to be shallow \( (\omega \sqrt{\frac{h}{g}} = kh \ll 1) \)